Throughout these templates, let $\sum_{n=1}^{\infty} a_n$ be a series. We hope to determine the convergence of this series.

Divergence Test: If $\lim_{n\to\infty} a_n$ is not zero or does not exist, then the series $\sum_{n=1}^{\infty} a_n$ diverges.

- Preliminary check: are the terms that we are adding up go to zero or not? If not, proceed! If the terms a_n are going to zero, pick another test.

- Computation: Find $\lim_{n\to\infty} a_n$.

- State the conclusion: Since $\lim_{n\to\infty} a_n \neq 0$ (or doesn't exist, whatever the case may be) by the divergence test, the series $\sum_{n=1}^{\infty} a_n$ diverges.

Integral Test: Suppose f is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$. Then the series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if the improper integral $\int_1^{\infty} f(x)\,dx$ is convergent.

- Preliminary check: If we replace the discrete index n with the continuous variable x in the expression of a_n, is the resulting function $f(x)$ (which satisfies $a_n = f(n)$) something we can integrate?

- Check the hypotheses:
  (a) Is $f(x)$ continuous on $[1, \infty)$?
  (b) Is $f(x)$ positive on $[1, \infty)$?
  (c) Is $f(x)$ decreasing on $[1, \infty)$? (This part usually requires a computation of a derivative.)

- Computation: Evaluate $\int_1^{\infty} f(x)\,dx$. Does this improper integral converge or diverge? (Remember we have tests for this too!)

- State the conclusion: Note that $f(x)$ is a continuous, positive, decreasing function on $[1, \infty)$ and $a_n = f(n)$. Since $\int_1^{\infty} f(x)\,dx$ is convergent (divergent), by the integral test, the series $\sum_{n=1}^{\infty} a_n$ is also convergent (divergent).

Comparison Test: Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with positive terms. If $\sum_{n=1}^{\infty} b_n$ is convergent and $a_n \leq b_n$ for all n, then $\sum_{n=1}^{\infty} a_n$ is also convergent. If $\sum_{n=1}^{\infty} b_n$ is divergent and $a_n \geq b_n$ for all n, then $\sum_{n=1}^{\infty} a_n$ is also divergent.

- Preliminary check: Write down the sequence b_n that you want to compare to. Does $\sum_{n=1}^{\infty} b_n$ converge or diverge? What is your justification?

- Check the hypotheses:
  (a) Are $a_n \geq 0$ and $b_n \geq 0$ (at least eventually)?
  (b) If $\sum_{n=1}^{\infty} b_n$ converges, we want to show $a_n \leq b_n$.
  If $\sum_{n=1}^{\infty} b_n$ diverges, we want to show $a_n \geq b_n$.

- Computation: Show the necessary inequality from above.

- State the conclusion: Since a_n and b_n are positive for all n, and $a_n \leq b_n$ ($a_n \geq b_n$) and $\sum_{n=1}^{\infty} b_n$ converges (diverges), then by the comparison test, $\sum_{n=1}^{\infty} a_n$ converges (diverges) also.

Limit Comparison Test: Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with positive terms. If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ where c is a finite number and $c > 0$, then either both series converge or both diverge.

- Preliminary check: Write down the sequence b_n that you want to compare to. Does $\sum_{n=1}^{\infty} b_n$ converge or diverge? What is your justification?

- Check the hypothesis: Are $a_n \geq 0$ and $b_n \geq 0$ (at least eventually)?

- Computation: Compute $\lim_{n\to\infty} \frac{a_n}{b_n}$ and confirm that this limit is a positive, finite number.

- State the conclusion: Since a_n and b_n are positive for all n, and $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ where c is a finite number and $c > 0$, and $\sum_{n=1}^{\infty} b_n$ converges (diverges), then by the limit comparison test, $\sum_{n=1}^{\infty} a_n$ converges (diverges) also.

Alternating Series Test: Suppose that we have an alternating series. That is $a_n = (-1)^n |a_n|$ or $a_n = (-1)^{n-1} |a_n|$ (assume $a_n > 0$). If $|a_{n+1}| \leq |a_n|$ for all n and $\lim_{n\to\infty} |a_n| = 0$, then the series $\sum_{n=1}^{\infty} a_n$ is convergent.

- Preliminary check: Is the series alternating?

- Check the hypotheses:
  (a) Is $|a_{n+1}| \leq |a_n|$ for all n? (Are the terms getting smaller in absolute value?)
  (b) Is $\lim_{n\to\infty} |a_n| = 0$?

- Computation: Show any necessary computations to verify the hypotheses above.

- State the conclusion: Since $\sum_{n=1}^{\infty} a_n$ is an alternating series and $|a_{n+1}| \leq |a_n|$ for all n and $\lim_{n\to\infty} |a_n| = 0$, then the series $\sum_{n=1}^{\infty} a_n$ is convergent by the alternating series test.

Ratio Test: Let $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L$.

(a) If $L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent (and therefore convergent).

(b) If $L > 1$, or $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

(c) If $L = 1$, the test in inconclusive. No conclusion can be made about the convergence or divergence of $\sum_{n=1}^{\infty} a_n$.

- Computation: Evaluate $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$. If it is finite, we usually call it L. As long as this limit is not 1, we can proceed to state a conclusion.

- State the conclusion: Since $L < 1$ ($L > 1$ or the limit is infinite), then by the ratio test, the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent and convergent (divergent).